

# CM-Based Blind Equalization for Time-Varying MIMO-FIR Channels with Single Pulsation

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**Abstract**— The constant modulus (CM) property of source signal can be mostly utilized to equalize the single- input multiple-output and finite impulse response (FIR) channels. In existing, the channels are blindly equalized and the equalization methods are based on higher order statistics that estimates all non-zero channel pulsation under time-invariance assumption. However, the assumption cannot be satisfied time varying multiple input multiple output (MIMO) applications ex. Mobile communication. In this paper, the proposed algorithm which extends the CM criterion to blind equalization using complex exponential basic expansion model (CE-BEM) and the channel is assumed as time varying MIMO-FIR. The method only employ the Second order statistics (SOS) and finally, it estimates only one pulsation. In this way, the system increases the SNR of the transmitted symbols and yield most beneficial result in time-varying channels. The fast convergence is also achieved through zero forcing equalization.

**Index Terms** — Blind equalization, CE-BEM, constant modulus criterion, MIMO, second order statistics, stochastic gradient algorithm, zero forcing equalization.

## 1 INTRODUCTION

Nowadays the blind equalization of MIMO FIR channels has received considerable research interests. The challenge is to achieve the equalization in time varying MIMO channels ex. Wireless communication without any performance degradation. The Inter symbol Interference (ISI) and channel noise may be a common problem faced in wireless and mobile communication. Traditionally, the conventional equalization technique is used to overcome the ISI. If the channel is unknown and/or time-varying, a fixed sequence of symbols (training sequence) may be transmitted for channel probing. In the receiver, based on the knowledge of the training sequence, the channel model is identified and/or an equalizer is designed accordingly. The drawback of the method is the usage of training sequence and the time-invariant assumption, which is too restrictive to be satisfied in many practical communication systems. For example, according to 900MHz GSM standard, 26-bits out of every 148-bit frames are used as training signals. Due to the avoidance of using a training sequence, blind methods have the potential to save frequency bandwidth, improve energy efficiency and increase system throughput. In blind channel equalization, the transmission channels are estimated and the transmitted source signal is recovered from the channel outputs only.

The Time-varying finite impulse response (FIR) channels are often used in several practical communication systems. Statistical modeling of the channel is well motivated when time-varying path delays arise due to a large number of scatter signals.

Deterministic basis expansion models (BEM) have gained popularity for wireless applications, especially when the multipath is caused by a few strong reflectors and path delays exhibit variations due to the mobiles. The time-varying taps are expressed as a superposition of time-varying bases (complex exponentials when modeling Doppler effects) with time in-variant coefficients. One of the most popular method of basic expansion model is CE-BEM. It has widely been used to describe the TV FIR channels [9]-[12]. Based on the CE-BEM, several algorithms are developed for the blind equalization of TV FIR channels with single input [9]-[12], among which the algorithms in [9]-[11] depend on the accurate knowledge of all nonzero channel pulsations. Although the channel pulsation in [9],[10],[11] can be estimated using Higher order statistics (HOS) and it requires source signal to have non-zero fourth order moment. As a result, they are costly in computation to get good estimation results and are not applicable to some classes of signals, e.g., 8 phase-shift keying (8-PSK) signals [9]. Recently, a second-order statistics (SOS)-based approach has been proposed to estimate the pulsations [13]. However, its computational complexity dramatically rises as the number of pulsations increases [1].

The constant modulus algorithm (CMA) is perhaps the most well-known blind algorithm and is used in many practical applications because it requires no carrier synchronization. The complex envelope is constant in many signals, such as FM, CPFSK modulation, and square pulse-shaped complex pulse amplitude modulation (PAM), and this property is called the constant modulus (CM) signal property which exploits the constant modularity of the transmitted signal for adapting the parameters of an equalizer. This paper deals with the direct blind equalization of a TV MIMO-FIR channel that is driven by a signal with a constant modulus (CM) constellation. The CM property of digitally modulated signal can be worked to

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recover source signals from their instantaneous mixtures [1]. Although large numbers of CM algorithms are proposed for blind channel equalization. For example, The CM approach has been recently applied to the blind frequency-offset estimator for Orthogonal Frequency Division Multiplex (OFDM) systems [2] and also the blind detection of code-division multiple access (CDMA) signals in multipath channels and blind equalization of non-reducible channels [3-8].

In general, the constant modulus algorithm (CMA) seeks to minimize a cost defined by the CM criterion. The CM criterion penalizes deviations in the modulus (i.e., magnitude) of the equalized signal away from a fixed value. In certain ideal conditions, minimizing the CM cost can be shown to result in perfect (zero-forcing) equalization of the received signal. Remarkably, the CM criterion can successfully equalize signals characterized by source alphabets not possessing a constant modulus [e.g., 16- quadrature amplitude modulation (QAM)], as well as those possessing a constant modulus (e.g., 8-PSK) (see Fig. 1).

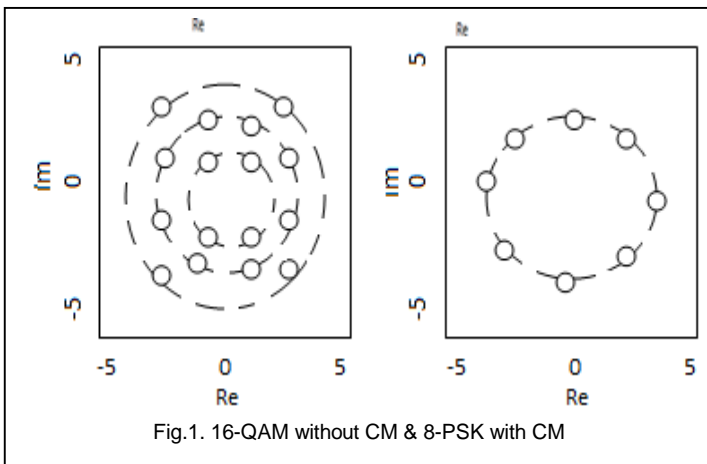


Fig.1. 16-QAM without CM & 8-PSK with CM

In this paper, based on the CE-BEM, the zero-forcing (ZF) equalization can be achieved using the CM criterion. Compared with the existing methods in [9]–[12], the proposed CM-based equalization method only needs to estimate one pulsation using SOS, resulting in a more accurate estimation for pulsations, and can be applied to a wider range of source signals due to the relaxation of constraints on the source signal.

## 2 CHANNEL MODEL

Consider a TV MIMO-FIR channel system with  $M$  inputs and  $M$  outputs. In this system, the output is given by

$$x_q(k) = \sum_{l=0}^L \sum_{q=0}^{Q-1} h_q(k; l) s_q(k-l) + v_q(k) \quad (1)$$

Where,  $x_q(k) = [x_1(k), x_2(k), \dots, x_M(k)]^T$  denotes the output vector and after stacking some successive observations of the channel outputs which yields

$$\bar{X}(k) = [X^T(k), X^T(k-1), \dots, X^T(k-K)]^T \quad (2)$$

$s_q(k) = [s_1(k), s_2(k), \dots, s_M(k)]^T$  is the source signal with CM constellation,  $v_q(k) = [v_1(k), v_2(k), \dots, v_M(k)]^T$  is the vector with  $M$  additive white Gaussian noise signals, independent of  $s(k)$ ,  $h_q(k; l) = [h_1(k; l), h_2(k; l), \dots, h_q(k; l)]^T$  is the time varying impulse response vector, and the subscript  $T$  stands for the transpose operation. In this paper, we propose the CE-BEM under the assumption of path delay vary with time linearly and the time varying channel impulse response  $h_q(k; l)$  in the CE-BEM are described as the finite linear combination of some complex exponential basis functions, say  $Q$ , i.e.  $\{e^{j\omega_q k}, e^{j\omega_2 k}, \dots, e^{j\omega_{Q-1} k}\}$  where  $k$  is the discrete time index and  $j = \sqrt{-1}$ .

$$h_p(k; l) = \sum_{q=0}^{Q-1} h_{p,q}(l) e^{j\omega_q k} \quad l = 0, 1, \dots, L$$

$$p = 1, 2, \dots, P \quad (3)$$

Where  $h_{p,q}(l)$  are some time-invariant coefficients and  $\omega_0, \omega_2, \dots, \omega_{Q-1}$  are distinct real numbers called pulsations (or basis frequencies). In (2), the channel variations are caused by the complex exponential basis function  $\{e^{j\omega_q k}\}_{q=0}^{Q-1}$  which are solely determined by the  $Q(Q \geq 2)$  distinct basis frequencies  $\{\omega_0, \omega_1, \dots, \omega_{Q-1}\}$  with  $\omega_0 < \omega_1 < \dots < \omega_{Q-1}$ .

From equation 1 & 2, it follows that

$$x_q(k) = \sum_{l=0}^L \sum_{q=0}^{Q-1} e^{j\omega_q k} h_{p,q}(k; l) s_q(k-l) + v_q(k) \quad (4)$$

Clearly the equation shows that, the recognition of time varying channels is equivalent to the estimation of the basis frequencies  $\{\omega_q\}_{q=0}^{Q-1}$  as well as the expansion coefficients  $h_{p,q}(l)$ .

To proceed, we define the  $M \times Q$  matrices  $H_n (n = 0, 1, \dots, L)$ , the  $M(K+1) \times Q(P+1)$  matrix, and the  $Q(P+1) \times Q(P+1)$  diagonal matrix  $C(k)$  as follows,

$$H_1 = \begin{bmatrix} e^{j\omega_1 l} h_{1,1}(l) & e^{j\omega_2 l} h_{1,2}(l) & \dots & e^{j\omega_{Q-1} l} h_{1,Q}(l) \\ e^{j\omega_1 l} h_{2,1}(l) & e^{j\omega_2 l} h_{2,2}(l) & \dots & e^{j\omega_{Q-1} l} h_{2,Q}(l) \\ \vdots & \vdots & \ddots & \vdots \\ e^{j\omega_1 l} h_{M,1}(l) & e^{j\omega_2 l} h_{M,2}(l) & \dots & e^{j\omega_{Q-1} l} h_{M,Q}(l) \end{bmatrix} \quad (5)$$

And

$$H = \begin{bmatrix} H_0 & \dots & H_L & \dots & 0_{M \times Q} \\ \vdots & \backslash & \backslash & \backslash & \vdots \\ 0_{M \times Q} & \dots & H_0 & \dots & H_L \end{bmatrix}$$

$$C(k) = \text{diag} \left( \underbrace{e^{j\omega_1 k}, \dots, e^{j\omega_q k}}_q, \dots, \underbrace{e^{j\omega_1 (k-P)}, \dots, e^{j\omega_q (k-P)}}_q \right) \quad (6)$$

Where  $P = K + L$ . The  $H$  is referring to toeplitz matrix and  $c(k)$  is a diagonal matrix. Aforementioned equations, we summarize the  $\bar{X}(k)$  as follows that

$$\bar{X}(k) = HC(k)\bar{s}(k) \quad (7)$$

The task here is to estimate the basic frequencies from second order statistics of the channel outputs  $x_1(k), x_2(k), \dots, x_q(k)$ . To proceed, we assume in the sequel that

A1) The source signal  $s_q(k)$  is a zero-mean white stationary stochastic process and has  $S = 2^R$  constellation points, which is uniformly distributed on the unit circle, where  $R \geq 2$  is a positive integer.

A2) The noise sequences  $v_q(k) = [v_1(k), v_2(k), \dots, v_q(k)]$  are stationary, temporally and spatially white, and independent of  $s_q(k)$ .

A3) The minimum basis frequency  $\omega_0 = 0$  and the maximum basis frequency is  $\omega_q$ . i.e.  $0 = \omega_0 < \omega_1 < \dots < \omega_q$ .

### 3 BLIND EQUALIZATION USING THE CONSTANT MODULUS CRITERION

Let  $g$  be an equalization vector and define

$$y(k) = g^T \bar{x}(k) \quad (8)$$

The zero-forcing equalizer can be developed with the equalization vector. i.e  $g$  satisfies the following condition

$$g^T H = [0, \dots, 0, \zeta, 0, \dots, 0] \quad \text{and} \quad \zeta = 1 \text{ (perfect equalization)} \quad (9)$$

To find the preferred equalization vector  $g$ , and show the CM criterion as follows,

$$J(g) = \frac{1}{4} E[|y(k)|^2 - 1]^2 \quad (10)$$

Generally, the stochastic gradient algorithm minimizes the CM cost function and has been developed as

$$g(k+1) = g(k) - \mu(|y(k)|^2 - 1)y(k)\bar{x}^*(k) \quad (11)$$

Where  $\mu$  is the step size and the superscript \* denotes the complex conjugate function. By proper tuning of the step size parameter, convergence usually occurs within a few iterations (between 5 and 10 iterations is typical). It has been established in [12] and [13].

Furthermore, we minimize the cost function by modifying the above algorithm as follows,

$$\min J = E[|g^H y(k)|^2 - 1] \quad (12)$$

The update rule is given by

$$g_{k+1} = g_k - \mu \cdot K[|g^H y(k)|^2 - 1]y(k)y^H(k)g_k \quad (13)$$

Here we assign the numerical constant  $K = 2$ . Then after we estimate the zero forcing equalization vector  $g$ . From that we compute the output vector  $y(k)$ . i.e

$$y(k) = g^T \bar{X}(k) \quad (14)$$

#### 3.1 ESTIMATION OF UNKNOWN PULSATION:

Furthermore we express the aforementioned equation (14) as

$$y(k) = g^T H C(k) \bar{s}(k) = \zeta s(k - t_0) e^{j\omega_q (k - t_0)} \quad (15)$$

Where  $\omega_q \in \{\omega_1, \omega_2, \dots, \omega_q\}$ , and  $0 \leq t_0 \leq P$ . Now the remaining task is estimation of unknown pulsation  $\omega_q$  and then determine the set  $\{\omega_q - \omega_q\}_q$  by using the Second-order statistics  $E[y(k) \cdot \sum_{t=0}^{k-1} x_p^*(k-t)]$ . Finally we recover the source signal  $s(k)$  by removing the  $e^{j\omega_q (k)}$  from  $y(k)$ . According to the assumption (3)  $\omega_0 = 0$  and the largest element in the set is  $\omega_q - \omega_0 = \omega_q$  in theory and is estimate of  $\omega_q$  in practice.

The summarization of CM based blind equalization algorithm is formulated as follows

- Step1. Generate the source signal  $s(k)$  which are uniformly distributed on the unit circle and then add white noise to that signal.
- Step2. Next Compute the equalizer output vector  $y(k) = g^T \bar{X}(k)$
- Step3. Compute the Fourier series  $E[y(k) \cdot \sum_{t=0}^{k-1} x_p^*(k-t)]$  and then determine the set  $\{\omega_q - \omega_q\}_{q=1}^Q$ .
- Step 4. Find the largest element in  $\{\omega_q - \omega_q\}_{q=1}^Q$  as the estimate of the frequency  $\omega_q$ .
- Step5. Estimate the source signal  $s(k)$  from  $y(k) \cdot e^{-j\omega_q k}$

Fig.3. shows the bit error rate performance (BER) of the proposed algorithm with the source of 8-PSK signal.

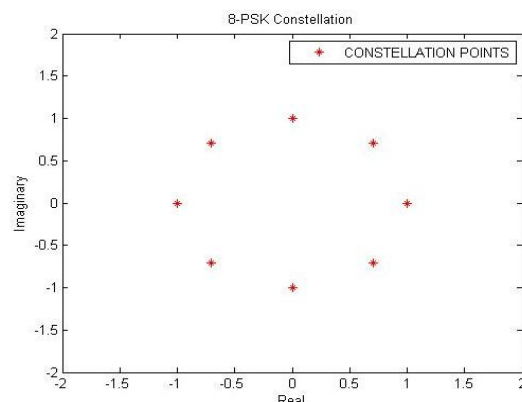


Fig.2 8-PSK source signal

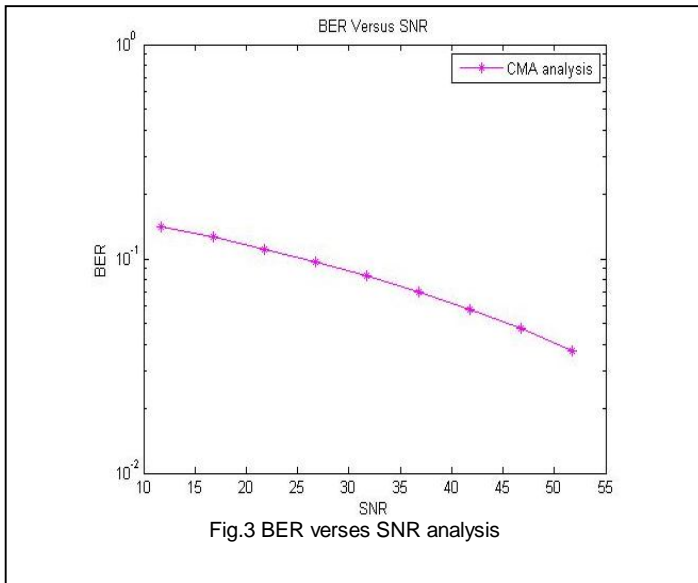
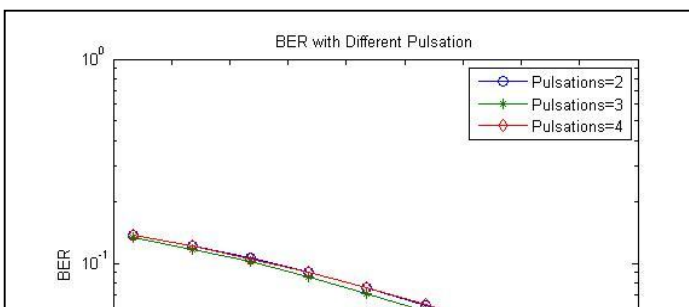


Fig.3 BER verses SNR analysis

#### 4 NUMERICAL SIMULATIONS

##### Example:

Let us consider the TV  $2 \times 4$  MIMO FIR channel that has four outputs i.e  $L = 2$  and  $M = 4$ . The time invariant channel coefficients are generated randomly and we choose that coefficient as  $K = 1$ . Fig.2 shows 8-PSK signal, it is used as the source signals  $s(k)$ . The blind equalization methods in (1), can works this signal with SIMO system but not in MIMO. Even most of the equalization algorithm restrict the PSK signals; refer [6] & [9]. The CM based algorithm can effectively deal the above mentioned signals. Here we assume 4 pulsations: 1)  $\omega_0 = 0$ ; 2)  $\omega_1 = 2\pi/200$ ; 3)  $\omega_2 = 2\pi/110$ ; 4)  $\omega_3 = 2\pi/40$ ; Fig.4. Shows the bit error rate (BER) of the proposed algorithm verses the signal-to-noise ratio (SNR) under 3 channel scenarios 1) two pulsation 2) three pulsation 3) four pulsation.



The unknown pulse  $\omega_q$  is estimated from the set  $\{\omega_q - \omega_q\}_{q=1}^Q$  by using the second order statistics. Fig.5. Shows the pulse estimation of the proposed CM based algorithm which determine the  $\omega_q$ .

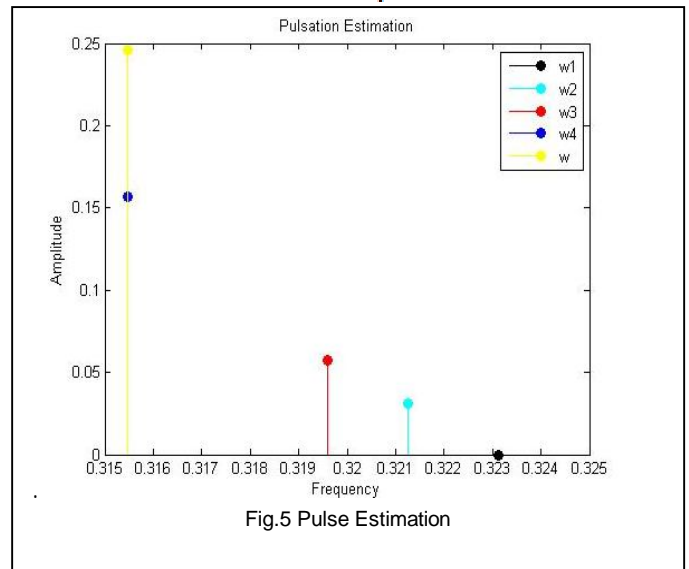


Fig.5 Pulse Estimation

#### 5 CONCLUSION

In this paper, the feasibility of utilizing the CM based blind equalization for TV MIMO channels are investigated. The proposed algorithm equalizes the TV MIMO channels and finally recovers the source signal that is described by CE-BEM. As a result, it can be applied to wide ranges of applications. Furthermore, the algorithm uses only the second-order statistics instead of higher order moments. Therefore, the computation complexity could be substantially reduced. This approach results in a more accurate estimation outcome. Simulation examples illustrate the good equalization performance of the CM based algorithm to the existing approach.

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